

C. U. SHAH UNIVERSITY

Winter Examination-2022

Subject Name: Engineering Mathematics - II

Subject Code: 4TE02EMT3

Branch: B.Tech (All)

Semester: 2

Date: 19/09/2022

Time: 11:00 To 02:00

Marks: 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

- a) The series $1 - \frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \dots + \infty$ is
 (a) convergent (b) divergent (c) finitely oscillating (d) infinitely oscillating
- b) Telescoping series is
 (a) convergent (b) divergent (c) finitely oscillating (d) None of these

c) $\int_0^{\pi/2} \sin^4 x \, dx = \underline{\hspace{2cm}}$
 (a) $\frac{3\pi}{16}$ (b) $\frac{6\pi}{16}$ (c) $\frac{9\pi}{16}$ (d) $\frac{3\pi}{4}$

- d) If equation of the curve is $y = f(x)$, then length of the curve is _____

(a) $s = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$ (b) $s = \int_{x_1}^{x_2} \sqrt{1 + \frac{dy}{dx}} \, dx$

(c) $s = \int_{x_1}^{x_2} \sqrt{1 - \left(\frac{dy}{dx}\right)^2} \, dx$ (d) None of these

- e) $\Gamma(n-1) = \underline{\hspace{2cm}}$. ($n \geq 2$)

(a) $(n-2)!$ (b) $(n-3)!$ (c) $(n-4)!$ (d) $(n-5)!$

f) $\beta\left(\frac{1}{2}, \frac{1}{2}\right) = \underline{\hspace{2cm}}$.

(a) $\sqrt{\pi}$ (b) 1 (c) 0 (d) π

- g) For $m > 0$, Gamma Function is _____

(a) convergent (b) divergent (c) finitely oscillating (d) None of these



- h)** Area of Region R is _____
 (a) $\int_R x \, dy$ (b) $\int_R y \, dx$ (c) $\iint_R dx \, dy$ (d) $\iiint_R dx \, dy \, dz$
- i)** Tangents to the curve at infinity are called _____.
 (a) Cusp (b) Node (c) Conjugate Point (d) Asymptotes
- j)** The transformation $x = r \cos \theta$, $y = r \sin \theta$ transform the area element $dydx$ into $|J|drd\theta$ where $|J|$ is equal to
 (a) r (b) r^2 (c) $r^2 \sin \phi$ (d) $r^2 \cos \phi$
- k)** The tangents are real and distinct then the double point is called _____.
 (a) Cusp (b) Node (c) Conjugate Point (d) Asymptotes
- l)** Which of the following is true?
 (a) $\Gamma(m) = 2 \int_0^\infty e^{-x^2} \cdot x^{2m-1} dx$ (b) $B(m, n) = B(n, m)$
 (c) $\Gamma\left(m + \frac{1}{2}\right) = \frac{2m-1}{2} \frac{2m-3}{2} \dots \frac{1}{2} \sqrt{\pi}$ (d) All are true
- m)** Degree of differential Equation $\frac{d^2 y}{dx^2} = \left(\frac{d^2 y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + y^2\right)^{\frac{1}{4}}$ is _____.
 (a) 2 (b) 3 (c) 4 (d) 1
- n)** What is the general solution of $xdy + ydx = 0$?
 (a) $xy = c$ (b) $x=cy$ (c) $y = cx$ (d) $xy = 0$

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

- a).** Prove that $\int_0^1 x^5(1-x^5)^{10} dx = \frac{1}{3} B(2,11)$. (05)
- b).** Evaluate: $\int_0^1 x^m \left(\log \frac{1}{x}\right)^n dx$ (05)
- c).** Show that $B(m, n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx$ (04)

Q-3 Attempt all questions (14)

- a).** Show that the series $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{n(n+1)} + \dots \infty$ converges and find its sum. (05)
- b).** Examine the series $\sum_{n=1}^{\infty} \frac{x^n}{n^p}$ for convergence using root test. (05)



c). Find radius of convergence for the series $x - \frac{x^3}{3} + \frac{x^5}{5} - \dots$ (04)

Q-4 Attempt all questions (14)

a). Solve: $\frac{dy}{dx} + 2y \tan x = \sin x$ given that $y = 0$ when $x = \frac{\pi}{3}$. (05)

b). Solve: $(x^2 + y^2 + 1)dx - 2xy dy = 0$. (05)

c). Solve: $\frac{dy}{dx} = \cos x \cos y - \sin x \sin y$. (04)

Q-5 Attempt all questions (14)

a). Transform the integral $\int_0^{4a} \int_{\frac{y^2}{4a}}^y \frac{x^2 - y^2}{x^2 + y^2} dx dy$ by changing to polar (05)

Co-ordinates and hence evaluate it.

b). Find the area between one arc of the cycloid $x = a(\theta + \sin \theta)$,
 $y = a(1 - \cos \theta)$ and its base. (05)

c). Evaluate: $\int_{-c}^c \int_{-b}^b \int_{-a}^a (x^2 + y^2 + z^2) dz dy dx$ (04)

Q-6 Attempt all questions (14)

a). Using reduction formula Evaluate $\int_0^{\pi} x \sin^7 x \cos^4 x dx$ (05)

b). Using reduction formula prove that $\int_0^1 x^m (1-x)^n dx = \frac{m! n!}{(m+n+1)!}$ (05)

c). Test the convergence of the integral $\int_0^{\infty} \frac{\cos x}{1+x^2} dx$ (04)

Q-7 Attempt all questions (14)

a). A circuit containing a resistance R , an inductance L in series is acted on by periodic electromotive force $E \sin \omega t$. If $i = 0$ at $t = 0$ show that (07)

$$i(t) = \frac{E}{\sqrt{R^2 + L^2 \omega^2}} \left[\sin(\omega t - \phi) + e^{-\frac{Rt}{L}} \sin \phi \right], \text{ where } \phi = \tan^{-1} \left(\frac{L\omega}{R} \right)$$

b). State and prove Legendre's formula. (07)

Q-8 Attempt all questions (14)

a). Find the volume of the solid generated by the revolution of the loop of the curve $x(x^2 + y^2) = a(x^2 - y^2)$ about the x - axis. (05)



b). Test the convergence of the integral $\int_0^{\infty} \frac{x^2}{\sqrt{x^5 + 1}} dx$ (05)

c). Trace the curve $y^2 = \frac{x^2(a - x)}{(a + x)}$; where $a > 0$. (04)

